

Superspace Gauge Fixing in Yang-Mills Matter Coupled Conformal Supergravity

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Abstract

In $D = 4$, $\mathcal{N} = 1$ conformal superspace, the Yang-Mills matter coupled supergravity system is constructed where the Yang-Mills gauge interaction is introduced by extending the superconformal group to include the Kähler isometry group of chiral matter fields. There are two gauge-fixing procedures to get to the component Poincaré supergravity: one via the superconformal component formalism and the other via the Poincaré superspace formalism. These two types of superconformal gauge-fixing conditions are analyzed in detail and their correspondence is clarified.

1 Introduction

In our previous paper [1] we have demonstrated the equivalence between the conventional component approach [2]-[9] as known as the superconformal tensor calculus and the recent superspace approach [10, 11] to $D = 4$, $\mathcal{N} = 1$ conformal supergravity (SUGRA). The detailed correspondences between two approaches were explicitly shown for superconformal gauge fields, curvatures and curvature constraints, general conformal multiplets and their transformation laws. We also briefly discussed the superconformal gauge fixing which leads to the Poincaré SUGRA.

The previous analysis was not sufficient in two points. One is that we have confined ourselves to the matter coupled SUGRA system in which there is no Yang-Mills (YM) gauge field of internal symmetry. Such YM interactions should generally be introduced by gauging the isometry of the Kähler manifold of chiral matter fields, which requires some extra work in conformal superspace approach. This way of including YM interactions in superspace has essentially been known for the super-Poincaré case [12]-[14], but not for the superconformal case.¹ As we will show in section 2, that is a simpler task in conformal superspace than in the super-Poincaré case, thanks to the simplicity of the algebra for covariant derivatives.

The isometry transformation of the Kähler manifold does not necessarily leave the Kähler potential invariant but induces the so-called Kähler transformation, i.e., a shift by holomorphic and anti-holomorphic functions. In this case, the chiral compensator is also transformed under the YM gauge transformation [16, 9]. In section 3, we discuss the superconformal gauge fixing in superspace for the YM matter coupled SUGRA system. Two types of gauge fixing are studied for realizing the canonically normalized Einstein-Hilbert (EH) and Rarita-Schwinger (RS) terms: one is applicable for non-vanishing superpotential and the other is independent of the superpotential term. For the former case, the superconformal gauge-fixing condition is YM gauge invariant so that the previous results in Ref. [1] almost hold. For the latter case, however, the gauge-fixing condition is not YM gauge invariant, which causes a modification of the YM gauge transformation and the covariant derivatives in the resultant Poincaré superspace.

Another insufficient point is that, while we have seen the correspondence between the two approaches on many quantities, we have not directly touched on the fact that the superspace formalism has much more gauge freedom and gauge fields than the component formalism. In superspace, the gauge fields and the gauge transformation parameters are superfields with higher θ components, and further the gauge superfields contain the spinor-indexed components, all of which do not appear in component approach. In section 4, we discuss how these extra degrees of freedom are fixed in order to have the component formalism. We explicitly give all the necessary gauge-fixing conditions component-field-wise so as to fix the higher θ components of superfield gauge transformation parameters. This gauge fixing from conformal superspace to superconformal component approach is depicted as the route I in Fig. 1. We show that the resultant theory after the gauge fixing agrees with the superconformal tensor calculus, that is, all the extra gauge fields are fixed to zero or reduced

¹For some special cases, YM interactions in conformal superspace were discussed, e.g., for the linear compensator [11] and for the pure Fayet-Iliopoulos $U(1)$ system [15].

superspace approach

component approach

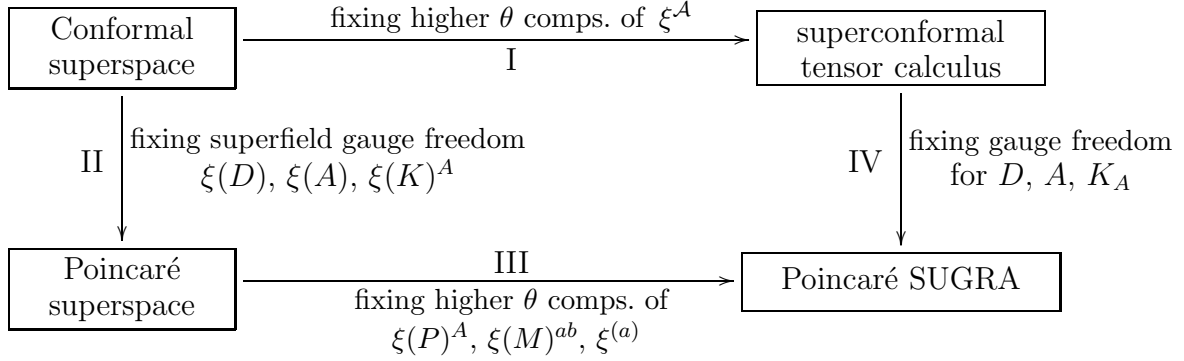


Figure 1: Relations among four SUGRA formulations: superspace and component approaches possessing superconformal and super-Poincaré gauge symmetries. There are two routes I+IV and II+III for getting to the Poincaré SUGRA from the conformal superspace formulation. The symbol ξ denotes the gauge transformation parameters of the superconformal and internal symmetries (see the text for details).

to the known quantities in component approach.

On the other hand, the gauge fixing discussed in section 3 corresponds to the route II in Fig. 1, where the gauge fixing is given superfield-wise to go down to the Poincaré superspace. The obtained Poincaré superspace formulation still has the gauge invariance with superfield gauge transformation parameters and their higher θ gauge invariance should be fixed to have the component Poincaré SUGRA. Such gauge fixing, the route III in Fig. 1, can be done in the same way as the superconformal case (the route I) that is clear from the discussion in section 4.

In section 5, we clarify in more detail how some type of superspace gauge fixing (the route II) corresponds to the so-called improved superconformal gauge in component approach (the route IV). It is noticed that there is a small puzzle: The component formulation seems to be obtained from the superspace one by setting the spinor-indexed components of gauge fields to zero. Nevertheless, in the Poincaré SUGRA obtained via the route II+III, non-vanishing A - and K_A -gauge fields with spinor indices remain and show up in the A - and K_A -gauge transformation parts in the Poincaré supersymmetry transformation. We give an answer to this in view of the resetting of gauge-fixing conditions.

2 YM matter coupled SUGRA in conformal superspace

In this section, we introduce the YM system in conformal superspace. The YM system is coupled to matter fields as an internal gauge symmetry. The internal gauge symmetry is the isometry of the Kähler manifold spanned by chiral matter fields, which isometry is generally given by nonlinear transformation. We use the so-called covariant approach in which one extends the superconformal covariant derivatives to be also covariant under the internal gauge symmetry [17, 14]. One advantage of the covariant approach is that the gauge transformation parameters are taken to be real (general) superfields, and hence the internal gauge symmetry is made manifest.

Then we present two types of superconformal gauge-fixing conditions which realize the canonically normalized EH and RS terms. One gives a real gravitino mass parameter and is adopted only when the superpotential does not vanish. The other can be imposed even when the superpotential vanishes but leads to a complex gravitino mass.

We consider the Lie algebra of a compact Lie group \mathcal{G} as an internal symmetry. The elements of the Lie algebra are denoted by $X_{(a)}$ where $(a) = 1, 2, \dots, \dim \mathcal{G}$. Their commutation relation is

$$[X_{(a)}, X_{(b)}] = -f_{(a)(b)}^{(c)} X_{(c)}, \quad (2.1)$$

where $f_{(a)(b)}^{(c)}$ is the structure constant of the Lie algebra. Since the spacetime symmetry and the internal symmetry are mutually independent, the elements of these two algebras commute with each other. We introduce the real gauge superfield $\mathcal{A}_M^{(a)}$ for the internal symmetry. In the following, we deal with the superconformal and internal symmetries simultaneously and denote all the elements collectively by $X_{\mathcal{A}}$. The gauge superfields and parameter superfields are denoted as

$$\begin{aligned} h_M^{\mathcal{A}} X_{\mathcal{A}} &= E_M^{\mathcal{A}} P_{\mathcal{A}} + \frac{1}{2} \phi_M^{ba} M_{ab} + B_M D + A_M A + f_M^{\mathcal{A}} K_{\mathcal{A}} + \mathcal{A}_M^{(a)} X_{(a)}, \\ \xi^{\mathcal{A}} X_{\mathcal{A}} &= \xi(P)^{\mathcal{A}} P_{\mathcal{A}} + \frac{1}{2} \xi(M)^{ba} M_{ab} + \xi(D) D + \xi(A) A + \xi(K)^{\mathcal{A}} K_{\mathcal{A}} + \xi^{(a)} X_{(a)}. \end{aligned} \quad (2.2)$$

Here, $E_M^{\mathcal{A}}$ is the vielbein superfield corresponding to the translation and supersymmetry generators $P_{\mathcal{A}} = (P_a, Q_{\alpha}, \bar{Q}^{\dot{\alpha}})$, ϕ_M^{ba} is the spin connection corresponding to the Lorentz generator M_{ab} , and $B_M, A_M, f_M^{\mathcal{A}}$ are the gauge superfields corresponding to the dilatation D , the $U(1)$ chiral transformation A , the conformal boost and its supersymmetry $K_{\mathcal{A}} = (K_a, S_{\alpha}, \bar{S}^{\dot{\alpha}})$, respectively. The gauge transformation parameter $\xi^{\mathcal{A}}$ is a real superfield. The deformed $P_{\mathcal{A}}$ transformation and covariant derivatives are defined by the general coordinate transformation δ_{GC} and the gauge transformation δ_G as

$$\begin{aligned} \delta_G(\xi(P)^{\mathcal{A}} P_{\mathcal{A}}) &= \delta_{GC}(\xi^{\mathcal{A}} E_{\mathcal{A}}^{\mathcal{M}}) - \delta_G(\xi^{\mathcal{B}} E_{\mathcal{B}}^{\mathcal{M}} h_M^{\mathcal{A}'} X_{\mathcal{A}'}), \\ \nabla_M &= \partial_M - \frac{1}{2} \phi_M^{ba} M_{ab} - B_M D - A_M A - f_M^{\mathcal{A}} K_{\mathcal{A}} - \mathcal{A}_M^{(a)} X_{(a)}. \end{aligned} \quad (2.3)$$

Here we set the parameter $\xi(P)^{\mathcal{A}}$ to be field-independent. The symbol $X_{\mathcal{A}'}$ means the generators other than $P_{\mathcal{A}}$. The gauge superfield $\mathcal{A}_M^{(a)}$ is transformed under the superconformal

and internal symmetries as

$$\delta(\xi^A X_A) \mathcal{A}_M^{(a)} = \partial_M \xi^{(a)} + \mathcal{A}_M^{(b)} \xi^{(c)} f_{(c)(b)}^{(a)} + E_M{}^B \xi(P)^C \mathcal{F}_{CB}^{(a)}, \quad (2.4)$$

where $\mathcal{F}_{MN}^{(a)}$ is the curvature superfield for the internal symmetry:

$$\mathcal{F}_{MN}^{(a)} = \partial_M \mathcal{A}_N^{(a)} - \partial_N \mathcal{A}_M^{(a)} - \mathcal{A}_M^{(b)} \mathcal{A}_N^{(c)} f_{(c)(b)}^{(a)}. \quad (2.5)$$

Similarly to the case without YM, we impose the curvature constraints $\{\nabla_\alpha, \nabla_\beta\} = 0$, $\{\bar{\nabla}_{\dot{\alpha}}, \bar{\nabla}_{\dot{\beta}}\} = 0$, and $\{\nabla_\alpha, \bar{\nabla}_{\dot{\beta}}\} = -2i\nabla_{\alpha\dot{\beta}}$, which implies $\mathcal{F}_{\alpha\beta}^{(a)} = \mathcal{F}_{\dot{\alpha}\dot{\beta}}^{(a)} = \mathcal{F}_{\alpha\dot{\beta}}^{(a)} = 0$ in the YM part. Solving the Bianchi identities under these constraints, we find that the curvatures $R_{\underline{\alpha}b}$ and R_{ab} can be expressed by a single “gaugino” superfield $\mathcal{W}_{\underline{\alpha}}$ as follows ($\underline{\alpha} = (\alpha, \dot{\alpha})$)

$$R_{\alpha, \beta\dot{\gamma}} = -[\nabla_\alpha, \nabla_{\beta\dot{\gamma}}] = 2i\epsilon_{\alpha\beta} \mathcal{W}_{\dot{\gamma}}, \quad R_{\dot{\alpha}, \beta\dot{\gamma}} = -[\bar{\nabla}_{\dot{\alpha}}, \nabla_{\beta\dot{\gamma}}] = 2i\epsilon_{\dot{\alpha}\beta} \mathcal{W}_{\dot{\gamma}}, \quad (2.6)$$

$$R_{\alpha\dot{\alpha}, \beta\dot{\beta}} = -[\nabla_{\alpha\dot{\alpha}}, \nabla_{\beta\dot{\beta}}] = -\epsilon_{\dot{\alpha}\dot{\beta}} \{\nabla_{(\alpha}, \mathcal{W}_{\beta)}\} - \epsilon_{\alpha\beta} \{\bar{\nabla}_{(\dot{\alpha}}, \mathcal{W}_{\dot{\beta})}\}, \quad (2.7)$$

and $\mathcal{W}_{\underline{\alpha}}$ contains the YM gaugino superfield $\mathcal{W}_{\underline{\alpha}}^{(a)}$:

$$\mathcal{W}_\alpha = (\epsilon\sigma^{bc})^{\beta\gamma} W_{\alpha\beta\gamma} M_{cb} + \frac{1}{2} \nabla^\gamma W_{\gamma\alpha}{}^\beta S_\beta - \frac{1}{2} \nabla^{\gamma\dot{\beta}} W_{\gamma\alpha}{}^\beta K_{\beta\dot{\beta}} + \mathcal{W}_\alpha^{(a)} X_{(a)}, \quad (2.8)$$

$$\mathcal{W}^{\dot{\alpha}} = (\bar{\sigma}^{bc}\epsilon)^{\dot{\gamma}\dot{\beta}} W_{\dot{\beta}\dot{\gamma}}{}^{\dot{\alpha}} M_{cb} - \frac{1}{2} \bar{\nabla}_{\dot{\gamma}} W^{\dot{\gamma}\dot{\alpha}}{}_{\dot{\beta}} \bar{S}^{\dot{\beta}} - \frac{1}{2} \nabla^{\dot{\gamma}\beta} W_{\dot{\gamma}}{}^{\dot{\alpha}\beta} K_{\beta\dot{\beta}} + \mathcal{W}^{(a)\dot{\alpha}} X_{(a)}. \quad (2.9)$$

Eq. (2.6) implies in the YM part

$$\mathcal{F}_{\alpha, \beta\dot{\gamma}}^{(a)} = 2i\epsilon_{\alpha\beta} \mathcal{W}_{\dot{\gamma}}^{(a)}, \quad \mathcal{F}_{\dot{\alpha}, \gamma\dot{\beta}}^{(a)} = 2i\epsilon_{\dot{\alpha}\gamma} \mathcal{W}_{\dot{\beta}}^{(a)}, \quad (2.10)$$

$$\mathcal{F}_{\alpha\dot{\alpha}, \beta\dot{\beta}}^{(a)} = -\epsilon_{\dot{\alpha}\dot{\beta}} \nabla_{(\alpha} \mathcal{W}_{\beta)}^{(a)} - \epsilon_{\alpha\beta} \bar{\nabla}_{(\dot{\alpha}} \mathcal{W}_{\dot{\beta})}^{(a)}. \quad (2.11)$$

The gaugino superfield $\mathcal{W}_{\underline{\alpha}}$, particularly $\mathcal{W}_{\underline{\alpha}}^{(a)}$ is found to satisfy the following superconformal property from the Bianchi/Jacobi identities,

$$\nabla^\alpha \mathcal{W}_\alpha^{(a)} = \bar{\nabla}_{\dot{\alpha}} \mathcal{W}^{\dot{\alpha}(a)}, \quad (2.12)$$

$$\bar{\nabla}_{\dot{\alpha}} \mathcal{W}_\alpha^{(a)} = 0, \quad D\mathcal{W}_\alpha^{(a)} = \frac{3}{2} \mathcal{W}_\alpha^{(a)}, \quad A\mathcal{W}_\alpha^{(a)} = i\mathcal{W}_\alpha^{(a)}, \quad K_A \mathcal{W}_\alpha^{(a)} = 0. \quad (2.13)$$

That is, $\mathcal{W}_\alpha^{(a)}$ is a covariantly chiral and primary superfield carrying the Weyl weight Δ and the chiral weight w with $(\Delta, w) = (3/2, 1)$. Note that $(\mathcal{W}_\alpha^{(a)})^\dagger = -\mathcal{W}_{\dot{\alpha}}^{(a)}$, and our $\mathcal{A}_M^{(a)}$ and $\mathcal{W}_\alpha^{(a)}$ are equivalent to $-i\mathcal{A}_M^{(r)}$ and $-i\mathcal{W}_\alpha^{(r)}$ in [14], respectively.

The coupling of YM to matter superfields in conformal superspace can be discussed in a similar way as in the component approach [9]. The matter primary superfields Φ^i ($i = 1, 2, \dots, n$) have the Weyl and chiral weights $(\Delta, w) = (0, 0)$, and are covariantly chiral with respect to the superconformal and internal symmetries:

$$\bar{\nabla}^{\dot{\alpha}} \Phi^i = 0. \quad (2.14)$$

The internal symmetry preserves the metric of the Kähler manifold spanned by chiral matter fields and their conjugates. The metric of the manifold is written by

$$g_{ij^*} = \frac{\partial^2 K}{\partial \Phi^i \partial \bar{\Phi}^{j^*}}, \quad (2.15)$$

where the Kähler potential K is a real function of Φ^i and $\bar{\Phi}^{i^*}$.

The generator $X_{(a)}$ acts on the matter superfields as a vector superfield $V_{(a)}$, which can be decomposed into the holomorphic part $V_{(a)}^-$ and anti-holomorphic one $V_{(a)}^+$:

$$V_{(a)} = V_{(a)}^- + V_{(a)}^+, \quad V_{(a)}^- = V_{(a)}^i(\Phi) \frac{\partial}{\partial \Phi^i}, \quad V_{(a)}^+ = \bar{V}_{(a)}^{i^*}(\bar{\Phi}) \frac{\partial}{\partial \bar{\Phi}^{i^*}}. \quad (2.16)$$

Thus, $X_{(a)}$ acts as

$$X_{(a)} \Phi^i = V_{(a)}^i(\Phi), \quad X_{(a)} \bar{\Phi}^{i^*} = \bar{V}_{(a)}^{i^*}(\bar{\Phi}), \quad (2.17)$$

and preserves the metric of the manifold, which means $V_{(a)}$ is the Killing vector:

$$V_{(a)}^i \frac{\partial g_{jk^*}}{\partial \Phi^i} + \frac{\partial V_{(a)}^i}{\partial \Phi^j} g_{ik^*} + \bar{V}_{(a)}^{i^*} \frac{\partial g_{jk^*}}{\partial \bar{\Phi}^{i^*}} + \frac{\partial \bar{V}_{(a)}^{i^*}}{\partial \bar{\Phi}^{k^*}} g_{ji^*} = 0. \quad (2.18)$$

Solving this equation, the action of $V_{(a)}^\pm$ on the Kähler potential is found to be expressed as

$$V_{(a)}^i K_i = F_{(a)} - iJ_{(a)}, \quad \bar{V}_{(a)}^{i^*} K_{i^*} = \bar{F}_{(a)} + iJ_{(a)}, \quad (2.19)$$

where $K_i = \partial K / \partial \Phi^i$ and $K_{i^*} = \partial K / \partial \bar{\Phi}^{i^*}$. On the RHS, $F_{(a)}$ is a holomorphic function and $J_{(a)}$ is a real function called the Killing potential or moment map.

The superspace action of YM matter coupled conformal SUGRA is given by

$$S = -3 \int d^4x d^4\theta E \Phi^c \bar{\Phi}^c e^{-K/3} + \left(\int d^4x d^2\theta \mathcal{E} (\Phi^c)^3 W(\Phi) - \frac{1}{4} \int d^4x d^2\theta \mathcal{E} H_{(a)(b)}(\Phi) \mathcal{W}^{\alpha(a)} \mathcal{W}_\alpha^{(b)} + \text{h.c.} \right). \quad (2.20)$$

Here the chiral superfield Φ^c , called the chiral compensator, is primary and has the weights $(\Delta, w) = (1, 2/3)$. The superpotential $W(\Phi)$ and the gauge holomorphic function $H_{(a)(b)}(\Phi)$ are holomorphic functions of matter superfields Φ^i , and hence primary chiral superfields with vanishing weights $(\Delta, w) = (0, 0)$. The indices of $H_{(a)(b)}$ are symmetric under the exchange $(a) \leftrightarrow (b)$. If we require the gauge invariance under the internal symmetry, $X_{(a)}$ should act on the chiral compensator, the superpotential and the gauge holomorphic function as

$$X_{(a)} \Phi^c = \frac{1}{3} F_{(a)} \Phi^c, \quad X_{(a)} \bar{\Phi}^c = \frac{1}{3} \bar{F}_{(a)} \bar{\Phi}^c, \quad X_{(a)} W = -F_{(a)} W, \quad (2.21)$$

$$X_{(a)} H_{(b)(c)} = V_{(a)} H_{(b)(c)} = -f_{(a)(b)}^{(d)} H_{(d)(c)} - f_{(a)(c)}^{(d)} H_{(b)(d)}.$$

3 Gauge fixing to Poincaré superspace

We first discuss the superconformal gauge-fixing condition for going down to the Poincaré superspace. For the case of non-vanishing superpotential, the chiral compensator Φ^c can be redefined as

$$\Phi^c \rightarrow \Phi^0 = \Phi^c W^{1/3}. \quad (3.1)$$

The new chiral compensator Φ^0 still has the weights $(\Delta, w) = (1, 2/3)$. The integrands of matter action become $\Phi^c \bar{\Phi}^c e^{-K/3} = \Phi^0 \bar{\Phi}^0 e^{-G/3}$ and $(\Phi^c)^3 W = (\Phi^0)^3$ with

$$G = K + \ln |W|^2. \quad (3.2)$$

Note that this redefinition is possible only when $W \neq 0$. The set of superconformal gauge-fixing conditions which realizes the canonically normalized EH and RS terms and also gives a real gravitino mass is

$$D, A \text{ gauge : } \Phi^0 = e^{G/6}, \quad K_A \text{ gauge : } B_M = 0. \quad (3.3)$$

One of the virtues of using Φ^0 and G is that they are invariant under the gauged internal symmetry: $X_{(a)}\Phi^0 = X_{(a)}G = 0$, which follow from Eqs. (2.19) and (2.21). This invariance property of Φ^0 and G makes it simpler to fix the superconformal gauge symmetry independently of the internal one. So we can simply extend the previous results of the system without YM [1].

The gauge condition $B_M = 0$ and the curvature constraint $R(D)_{\underline{a}B} = 0$ constrain the K_A -gauge superfields, similarly to the case without YM [10]. The following is the restricted form of K_A -gauge superfields which is needed for later discussion:

$$f_{\alpha\beta} = -f_{\beta\alpha} = -\epsilon_{\alpha\beta}\bar{R}, \quad f_{\dot{\alpha}\dot{\beta}} = -f_{\dot{\beta}\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}R, \quad f_{\alpha\dot{\beta}} = -f_{\dot{\beta}\alpha} = -\frac{1}{2}G_{\alpha\dot{\beta}}, \quad f_{\underline{a}b} = -f_{b\underline{a}}. \quad (3.4)$$

The equations $\bar{\nabla}^{\dot{\alpha}}\Phi^0 = 0$ and $\bar{\nabla}^{\dot{\alpha}}\nabla_{\dot{\beta}}\Phi^0 = -2i\nabla^{\dot{\alpha}}_{\dot{\beta}}\Phi^0$, which follow from the chirality condition of Φ^0 , determine the A -gauge fields in the same form as before [(4.12), (4.13) and (4.21) in Ref.[1]]. Under the gauge-fixing condition (3.3), we obtain

$$\begin{aligned} A_{\alpha} &= \frac{i}{4}G_j \mathcal{D}_{\alpha}^P \Phi^j, & A_{\dot{\alpha}} &= -\frac{i}{4}G_{j*} \bar{\mathcal{D}}_{\dot{\alpha}}^P \bar{\Phi}^{j*}, \\ A_{\alpha}{}^{\dot{\beta}} &= \frac{i}{4}(G_i \nabla_{\alpha}{}^{\dot{\beta}} \Phi^i - G_{i*} \nabla_{\alpha}{}^{\dot{\beta}} \bar{\Phi}^{i*}) + \frac{1}{4}G_{ij*} \nabla_{\alpha} \Phi^i \bar{\nabla}^{\dot{\beta}} \bar{\Phi}^{j*} - \frac{3}{2}G_{\alpha}{}^{\dot{\beta}}, \end{aligned} \quad (3.5)$$

where \mathcal{D}_A^P is the covariant derivative in the Poincaré SUGRA including the YM part and given by

$$\begin{aligned} \mathcal{D}_A^P &= E_A^M \partial_M - \frac{1}{2}\phi_A^{bc} M_{cb} - \mathcal{A}_A^{(a)} X_{(a)} \\ &= \nabla_A + A_A A + B_A D + f_A^B K_B. \end{aligned} \quad (3.6)$$

We further obtain the following expression for the components of chiral compensator Φ^0 [(4.18) and (4.19) in Ref. [1]]:

$$\nabla_\alpha \Phi^0| = \frac{1}{3} e^{G/6} G_i \nabla_\alpha \Phi^i|, \quad (3.7)$$

$$\nabla^2 \Phi^0| = \frac{1}{3} e^{G/6} \left(G_i \nabla^2 \Phi^i + (G_{ij} + \frac{1}{3} G_i G_j) \nabla^\alpha \Phi^j \nabla_\alpha \Phi^i - 24 \bar{R} \right)|. \quad (3.8)$$

Here the vertical bar “|” means the $\theta = \bar{\theta} = 0$ projection, i.e., the lowest component of superfield.

The gauge fixing (3.3) is the most convenient and physical one. But the redefinition (3.2) cannot be done when the system has no superpotential term. In this case, the most physical gauge fixing is given by

$$\Phi^c = e^{K/6}, \quad B_M = 0, \quad (3.9)$$

which also realizes the canonically normalized EH term (if $W \neq 0$, (3.9) generally leads to a complex gravitino mass). The main difference is that the gauge fixing (3.9) no longer preserves the internal gauge symmetry, since $e^{K/6}$ and Φ^c are transformed differently under $X_{(a)}$. This violation of internal symmetry can be compensated by the A -gauge transformation. Noting that $A\Phi^c = i\frac{2}{3}\Phi^c$ and $e^{K/6}$ is A -gauge invariant, we find that the following combination of the internal gauge and A transformations rotates Φ^c and $e^{K/6}$ in the same way:

$$\tilde{X}_{(a)} = X_{(a)} + \frac{i}{4} (F_{(a)} - \bar{F}_{(a)}) A. \quad (3.10)$$

Therefore $\tilde{X}_{(a)}$ gives the remaining internal gauge symmetry after the gauge fixing (3.9), and satisfies the same commutation relation as $X_{(a)}$

$$[\tilde{X}_{(a)}, \tilde{X}_{(b)}] = -f_{(a)(b)}^{(c)} \tilde{X}_{(c)}. \quad (3.11)$$

This is guaranteed by the relation $V_{(a)}^i \partial F_{(b)} / \partial \Phi^i - V_{(b)}^i \partial F_{(a)} / \partial \Phi^i = -f_{(a)(b)}^{(c)} F_{(c)}$, which is obtained by acting both sides of Eq. (2.1) on Φ^c . The A gauge field A_M is no longer inert under the internal gauge transformation $\tilde{X}_{(a)}$ but is transformed as

$$\delta_G(\xi^{(a)} \tilde{X}_{(a)}) A_M = \partial_M \left(\xi^{(a)} \frac{i}{4} (F_{(a)} - \bar{F}_{(a)}) \right), \quad (3.12)$$

which matches to the isometric Kähler transformation discussed in [14]. That is, from the superconformal viewpoint, the isometric Kähler transformation is understood to be the combination of internal gauge and A transformations which leaves the gauge-fixing condition (3.9) inert.

The covariant derivatives $\tilde{\mathcal{D}}_A^P$ in the Poincaré SUGRA after imposing (3.9) should be defined by using $\tilde{X}_{(a)}$ as

$$\tilde{\mathcal{D}}_A^P = E_A^M \partial_M - \frac{1}{2} \phi_A^{cb} M_{bc} - \mathcal{A}_A^{(a)} \tilde{X}_{(a)} \quad (3.13)$$

$$= \nabla_A + \left(A_A - \frac{i}{4} (F_{(a)} - \bar{F}_{(a)}) \mathcal{A}_A^{(a)} \right) A + B_A D + f_A^B K_B. \quad (3.14)$$

The A -gauge superfields and the components of chiral compensator can be obtained similarly to Eqs. (3.5), (3.7) and (3.8) as

$$\begin{aligned} A_\alpha &= \frac{i}{4} \left(K_j \tilde{\mathcal{D}}_\alpha^P \Phi^j + (F_{(a)} - \bar{F}_{(a)}) \mathcal{A}_\alpha^{(a)} \right), & A_{\dot{\alpha}} &= \frac{i}{4} \left(-K_{j*} \tilde{\mathcal{D}}_{\dot{\alpha}}^P \bar{\Phi}^{j*} + (F_{(a)} - \bar{F}_{(a)}) \mathcal{A}_{\dot{\alpha}}^{(a)} \right), \\ A_\alpha{}^{\dot{\beta}} &= \frac{i}{4} \left(K_i \tilde{\mathcal{D}}_\alpha^P \Phi^i - K_{i*} \tilde{\mathcal{D}}_\alpha^P \bar{\Phi}^{i*} + (F_{(a)} - \bar{F}_{(a)}) \mathcal{A}_\alpha^{(a)\dot{\beta}} \right) + \frac{1}{4} K_{ij*} (\tilde{\mathcal{D}}_\alpha^P \Phi^i) (\tilde{\mathcal{D}}^P{}^{\dot{\beta}} \bar{\Phi}^{j*}) - \frac{3}{2} G_\alpha{}^{\dot{\beta}}, \\ \nabla_\alpha \Phi^0 &= \frac{1}{3} e^{K/6} K_i \nabla_\alpha \Phi^i, & \nabla^2 \Phi^0 &= \frac{1}{3} e^{K/6} \left(K_i \nabla^2 \Phi^i + (K_{ij} + \frac{1}{3} K_i K_j) \nabla^\alpha \Phi^j \nabla_\alpha \Phi^i - 24 \bar{R} \right). \end{aligned} \quad (3.15)$$

The expressions for the A -gauge superfields are simply obtained from (3.5) by the replacement $A_A \rightarrow A_A - \frac{i}{4} (F_{(a)} - \bar{F}_{(a)}) \mathcal{A}_A^{(a)}$ as understood by the comparison of covariant derivatives. The gauge-fixed A -gauge superfields in (3.15) exactly agree with the composite gauge potential in the isometric Kähler superspace [14].

After the gauge fixing, we have the Poincaré SUGRA in superspace, which is the so-called isometric Kähler superspace. For the condition $\Phi^0 = e^{G/6}$, the gauge-fixed superspace action becomes

$$S = \int d^4x d^4\theta E \left(-\frac{3}{2} + \frac{e^{G/2}}{2R} - \frac{1}{8R} H_{(a)(b)} \mathcal{W}^{(a)\alpha} \mathcal{W}^{(b)}{}_\alpha \right) + \text{h.c.} . \quad (3.16)$$

Here we have used a relation of F and D-type actions

$$\int d^4x d^2\theta \mathcal{E} U = \int d^4x d^4\theta E \frac{U(\Phi^c \bar{\Phi}^c e^{-K/3})}{-\frac{1}{4} \bar{\nabla}^2 (\Phi^c \bar{\Phi}^c e^{-K/3})}, \quad (3.17)$$

where U is a chiral primary superfield with the weights $(\Delta, w) = (3, 2)$. Further, noticing that the gauge conditions (3.3) and their result for the K_A -gauge fields (3.4) lead to

$$-\frac{1}{4} \bar{\nabla}^2 (\Phi^c \bar{\Phi}^c e^{-K/3}) = f_{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\beta}\dot{\alpha}} (\Phi^c \bar{\Phi}^c e^{-K/3}) = 2R, \quad (3.18)$$

one finds that the action in conformal superspace (2.20) is reduced to (3.16). For the other gauge fixing $\Phi^c = e^{K/6}$, the resultant superspace action of Poincaré SUGRA is similarly obtained (by an apparent replacement $G \rightarrow K + \ln |W|^2$ in (3.16)).

4 Gauge fixing to component approach

Next we show the explicit form of superspace gauge fixing for going down to the superconformal tensor calculus in component approach. In conformal superspace, the gauge fields $h_M{}^A$ and the gauge transformation parameters ξ^A are superfields with higher θ components, and further the gauge superfields contain the spinor-indexed components $h_\mu{}^A$ and $h^{\dot{\mu}A}$ (which we call the spinor gauge superfields henceforth). But there is no such extra freedom of gauge fields in component approach. We show in the following that all these extra fields can be

gauge-fixed to be zero by using extra gauge freedom or otherwise be reduced to the known quantities in component approach.

Let us first remark that the following quantities in superspace have their counterparts in component approach so that they are known quantities:

1. The lowest component of curved vector gauge superfield, $h_m^{\mathcal{A}}|$. This corresponds to $h_\mu^{\mathcal{A}}$ in component approach.
2. The lowest components of flat spinor-spinor and spinor-vector curvatures, $R_{\underline{\alpha}\underline{\beta}}^{\mathcal{A}}|$ and $R_{a\underline{\beta}}^{\mathcal{A}}|$. These correspond to the coefficients of the terms appearing on the RHSs of the commutators $[\delta_Q, \delta_Q]$ and $[\delta_{\bar{P}}, \delta_Q]$ in component approach.
3. The lowest components of the covariant derivatives of flat spinor-spinor and vector-spinor curvatures $\nabla_{\underline{\beta}} \cdots \nabla_{\underline{\delta}} R_{\underline{\alpha}\underline{\beta}}^{\mathcal{A}}|$ and $\nabla_{\underline{\beta}} \cdots \nabla_{\underline{\delta}} R_{a\underline{\beta}}^{\mathcal{A}}|$. These correspond to the supersymmetry transformations of the coefficients in $[\delta_Q, \delta_Q]$ and $[\delta_{\bar{P}}, \delta_Q]$ in component approach.

We show that all other components in the θ expansions of spinor and vector gauge superfields can be gauge-fixed to zero or be written in terms of these known quantities.

4.1 Gauge conditions for spinor gauge superfields

The gauge transformation law of spinor gauge superfields is given by

$$\delta_G(\xi^B X_B) h_\mu^{\mathcal{A}} = \partial_\mu \xi^{\mathcal{A}} + h_\mu^{\mathcal{C}} \xi^B f_{BC}^{\mathcal{A}}. \quad (4.1)$$

The gauge-fixing procedure can be seen explicitly as follows. We first expand the gauge transformation parameter as

$$\begin{aligned} \xi^{\mathcal{A}}(x, \theta, \bar{\theta}) = & \xi^{(0,0)\mathcal{A}}(x) + \theta^\mu \xi^{(1,0)\mathcal{A}}_\mu(x) + \bar{\theta}_{\dot{\mu}} \bar{\xi}^{(0,1)\mathcal{A}\dot{\mu}}(x) \\ & + \theta^2 \xi^{(2,0)\mathcal{A}}(x) + \bar{\theta}^2 \bar{\xi}^{(0,2)\mathcal{A}}(x) + \theta^\mu \bar{\theta}_{\dot{\mu}} \xi^{(1,1)\mathcal{A}\dot{\mu}}_\mu(x) \\ & + \theta^2 \bar{\theta}^\mu \xi^{(2,1)\mathcal{A}}_\mu(x) + \bar{\theta}^2 \theta^\mu \xi^{(1,2)\mathcal{A}}_\mu(x) + \bar{\theta}^2 \theta^2 \xi^{(2,2)\mathcal{A}}(x). \end{aligned} \quad (4.2)$$

The lowest component $\xi^{(0,0)\mathcal{A}}(x)$ is identified with the usual gauge transformation parameter appearing in component approach. We use all the other higher θ transformation parameters $\xi^{(n,m)\mathcal{A}}(x)$ ($n + m \geq 1$) to fix the components of spinor gauge superfields:

	parameters	conditions on spinor gauge superfields
1st order	$\xi^{(1,0)\mathcal{A}}_\mu$	$E_\mu^{\mathcal{A}} = \delta_\mu^\alpha, \quad h_\mu^{\mathcal{A}'} = 0$
	$\xi^{(0,1)\mathcal{A}\dot{\mu}}$	$E^{\dot{\mu}\mathcal{A}} = \delta^{\dot{\mu}}_{\dot{\alpha}}, \quad h^{\dot{\mu}\mathcal{A}'} = 0$
2nd order	$\xi^{(2,0)\mathcal{A}}$	$\partial^\mu h_\mu^{\mathcal{A}} = 0$
	$\xi^{(0,2)\mathcal{A}}$	$\bar{\partial}_{\dot{\mu}} h^{\dot{\mu}\mathcal{A}} = 0$
	$\xi^{(1,1)\mathcal{A}\dot{\mu}}_\mu$	$\partial_\mu h^{\dot{\mu}\mathcal{A}} - \bar{\partial}^{\dot{\mu}} \partial^\mu h_\mu^{\mathcal{A}} = 0$
3rd order	$\xi^{(2,1)\mathcal{A}\dot{\mu}}$	$\partial^2 h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} \partial^\mu h_\mu^{\mathcal{A}} = 0$
	$\xi^{(1,2)\mathcal{A}}_\mu$	$\bar{\partial}^2 h_\mu^{\mathcal{A}} + \partial_\mu \bar{\partial}_{\dot{\mu}} h^{\dot{\mu}\mathcal{A}} = 0$
4th order	$\xi^{(2,2)\mathcal{A}}$	$\partial^2 \bar{\partial}_{\dot{\mu}} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^2 \partial^\mu h_\mu^{\mathcal{A}} = 0$

(4.3)

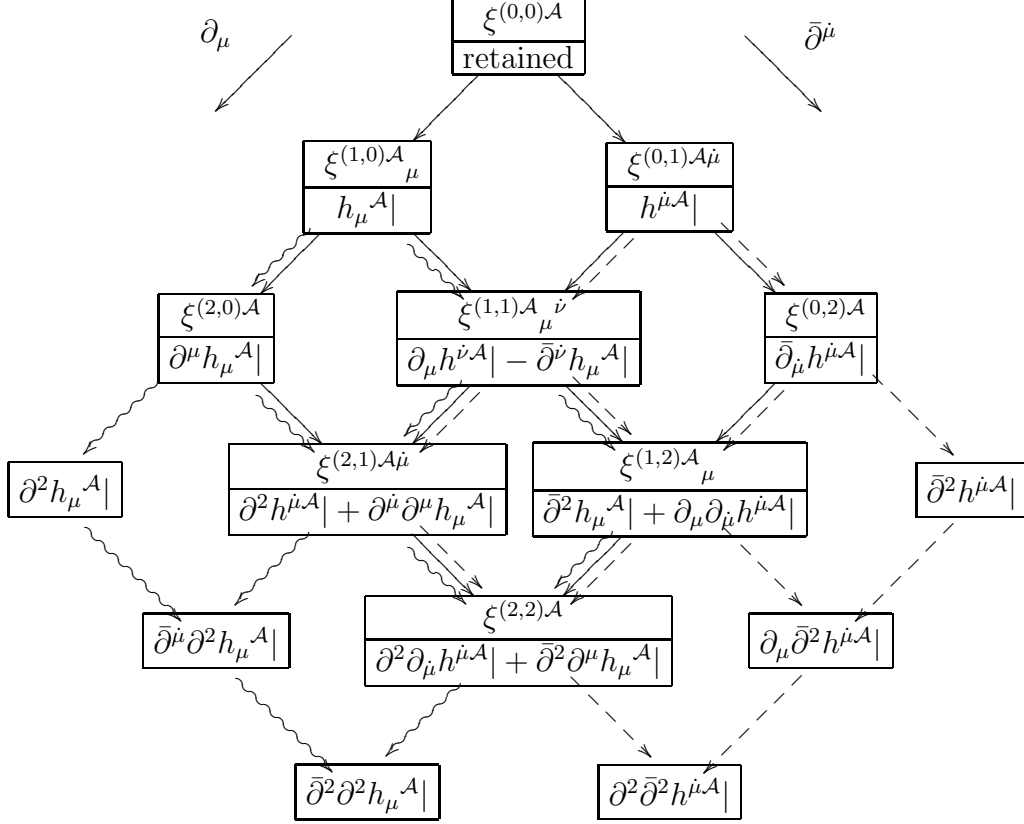


Figure 2: The 3×3 ‘diamond’ (denoted by rigid arrows) for the $\theta^n \bar{\theta}^m$ components $\xi^{(n,m)\mathcal{A}}$ ($n, m = 0, 1, 2$) of the gauge parameter superfield $\xi^{\mathcal{A}}$. The other two ‘diamonds’ (denoted by wavy and dashed arrows, respectively) for the $\theta^n \bar{\theta}^m$ components of the spinor gauge superfields $h_\mu^{\mathcal{A}}$ and $h^{\mu\mathcal{A}}$ are overlapped on it such that the gauge parameter $\xi^{(n,m)\mathcal{A}}$ at each point can be used to fix the spinor gauge field components on the same point. The explicit gauge field components which are gauge-fixed to zero (or constant) are shown at downstairs of the two-story boxes. The gauge field components lying outside of the gauge parameter diamond cannot be gauge-fixed but are determined by the curvature constraints as shown in the text.

This set of gauge-fixing conditions is visualized in Fig. 2. Note that the fixed gauge field components are indeed gauge-variant quantities which are shifted by the inhomogeneous transformation part $\delta h_\mu^{\mathcal{A}} = \partial_\mu \xi^{\mathcal{A}}$. For instance, using $\{\partial_\mu, \bar{\partial}^\mu\} = 0$, we have

$$-\varepsilon_{\mu\nu} \partial^\rho h_\rho^{\mathcal{A}} = \partial_\mu h_\nu^{\mathcal{A}} - \partial_\nu h_\mu^{\mathcal{A}}, \quad \partial^2 h^{\mu\mathcal{A}} + \bar{\partial}^\mu \partial^\mu h_\mu^{\mathcal{A}} = \partial^\mu (\partial_\mu h^{\mu\mathcal{A}} - \bar{\partial}^\mu h_\mu^{\mathcal{A}}). \quad (4.4)$$

We use the word “gauge-variant” when a quantity receives an inhomogeneous shift under the gauge transformations with $\xi^{(n,m)}(x)$ ($n + m \geq 1$), and otherwise call “gauge-invariant”. The point is that the gauge-fixed quantities in the above table exhaust the gauge variants. Therefore, once they are fixed to be zero (or constants), all the other quantities in the spinor gauge superfields $h_\mu^{\mathcal{A}}$ and $h^{\mu\mathcal{A}}$ can be expressed by gauge-invariant quantities, namely, by covariant curvatures.

Let us see this more explicitly. First we express all the curved spinor derivatives of the curved spinor gauge fields, $\partial_{\underline{\nu}} \cdots \partial_{\underline{\rho}} h_{\underline{\mu}}^{\mathcal{A}}$, in terms of the symmetric derivative

$$\partial_{\underline{\mu}} h_{\underline{\nu}}^{\mathcal{A}} + \partial_{\underline{\nu}} h_{\underline{\mu}}^{\mathcal{A}}, \quad (4.5)$$

which is later rewritten to the covariant curvature with flat spinor indices.

At the first order derivative level, setting the antisymmetric part equal to zero by the above gauge-fixing condition, we find

$$\partial_{\nu} h_{\mu}^{\mathcal{A}}| = \frac{1}{2}(\partial_{\nu} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h_{\nu}^{\mathcal{A}})|, \quad \bar{\partial}^{\nu} h^{\dot{\mu}\mathcal{A}}| = \frac{1}{2}(\bar{\partial}^{\nu} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h^{\nu\mathcal{A}})|, \quad (4.6)$$

$$\bar{\partial}^{\nu} h_{\mu}^{\mathcal{A}}| = \frac{1}{2}(\bar{\partial}^{\nu} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h^{\dot{\nu}\mathcal{A}})|, \quad \partial_{\nu} h^{\dot{\mu}\mathcal{A}}| = \frac{1}{2}(\partial_{\nu} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h_{\nu}^{\mathcal{A}})|. \quad (4.7)$$

At the second order derivative level, the identity $\partial_{\mu} \partial^{\nu} h_{\nu} = \frac{1}{2} \partial^2 h_{\mu}$ leads to

$$\partial^2 h_{\mu}^{\mathcal{A}}| = \frac{2}{3} \partial^{\nu} (\partial_{\nu} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h_{\nu}^{\mathcal{A}})|, \quad \bar{\partial}^2 h^{\dot{\mu}\mathcal{A}}| = \frac{2}{3} \bar{\partial}_{\dot{\nu}} (\bar{\partial}^{\dot{\nu}} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h^{\dot{\nu}\mathcal{A}})|, \quad (4.8)$$

which actually hold as the superfield equations without the vertical bars ‘|’. If we use the above gauge conditions by $\xi^{(2,1)\mathcal{A}\dot{\mu}}$ and $\xi^{(1,2)\mathcal{A}}_{\mu}$ we also have

$$\begin{aligned} \bar{\partial}^2 h_{\mu}^{\mathcal{A}} &= \frac{1}{2} \bar{\partial}_{\dot{\nu}} (\bar{\partial}^{\dot{\nu}} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h^{\dot{\nu}\mathcal{A}})|, & \partial^2 h^{\dot{\mu}\mathcal{A}}| &= \frac{1}{2} \partial^{\nu} (\partial_{\nu} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h_{\nu}^{\mathcal{A}})|, \\ \partial_{\rho} \bar{\partial}^{\nu} h_{\mu}^{\mathcal{A}}| &= \frac{1}{2} \partial_{\rho} (\bar{\partial}^{\nu} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h^{\dot{\nu}\mathcal{A}})| - \frac{1}{4} \bar{\partial}^{\nu} (\partial_{\rho} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h_{\rho}^{\mathcal{A}})|, \\ \bar{\partial}^{\dot{\rho}} \partial_{\nu} h^{\dot{\mu}\mathcal{A}}| &= \frac{1}{2} \bar{\partial}^{\dot{\rho}} (\partial_{\nu} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h_{\nu}^{\mathcal{A}})| - \frac{1}{4} \partial_{\nu} (\bar{\partial}^{\dot{\rho}} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h^{\dot{\rho}\mathcal{A}})|. \end{aligned} \quad (4.9)$$

At the third order derivative level, Eq. (4.8) without the bars leads to

$$\bar{\partial}^{\nu} \partial^2 h_{\mu}^{\mathcal{A}}| = \frac{2}{3} \bar{\partial}^{\nu} \partial^{\rho} (\partial_{\rho} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h_{\rho}^{\mathcal{A}})|, \quad \partial_{\nu} \bar{\partial}^2 h^{\dot{\mu}\mathcal{A}}| = \frac{2}{3} \partial_{\nu} \bar{\partial}_{\dot{\rho}} (\bar{\partial}^{\dot{\rho}} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h^{\dot{\rho}\mathcal{A}})|. \quad (4.10)$$

Using the gauge conditions by $\xi^{(2,2)}$, we have

$$\begin{aligned} \partial_{\nu} \bar{\partial}^2 h_{\mu}^{\mathcal{A}}| &= \frac{1}{2} \partial_{\nu} \bar{\partial}_{\dot{\rho}} (\bar{\partial}^{\dot{\rho}} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h^{\dot{\rho}\mathcal{A}})| + \frac{1}{4} \bar{\partial}^2 (\partial_{\nu} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h_{\nu}^{\mathcal{A}})|, \\ \bar{\partial}^{\dot{\nu}} \partial^2 h^{\dot{\mu}\mathcal{A}}| &= \frac{1}{2} \bar{\partial}^{\dot{\nu}} \partial^{\rho} (\partial_{\rho} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h_{\rho}^{\mathcal{A}})| + \frac{1}{4} \partial^2 (\bar{\partial}^{\dot{\nu}} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h^{\dot{\nu}\mathcal{A}})|. \end{aligned} \quad (4.11)$$

Finally at the fourth order level, Eq. (4.8) leads to

$$\bar{\partial}^2 \partial^2 h_{\mu}^{\mathcal{A}}| = \frac{2}{3} \bar{\partial}^2 \partial^{\nu} (\partial_{\nu} h_{\mu}^{\mathcal{A}} + \partial_{\mu} h_{\nu}^{\mathcal{A}})|, \quad \partial^2 \bar{\partial}^2 h^{\dot{\mu}\mathcal{A}}| = \frac{2}{3} \partial^2 \bar{\partial}_{\dot{\nu}} (\bar{\partial}^{\dot{\nu}} h^{\dot{\mu}\mathcal{A}} + \bar{\partial}^{\dot{\mu}} h^{\dot{\nu}\mathcal{A}})|. \quad (4.12)$$

In this way, all the curved spinor derivatives of the curved spinor gauge superfields are written in terms of the curved spinor derivatives on the symmetrized term $\partial_{\underline{\nu}} h_{\underline{\mu}}^{\mathcal{A}} + \partial_{\underline{\mu}} h_{\underline{\nu}}^{\mathcal{A}}$.

We show in the following that they are written by the flat-indexed curvatures $R_{CB}{}^A$ and their covariant spinor derivatives in the present gauge.

Noting the definition of curvatures

$$R_{MN}{}^A = \partial_M h_N{}^A - \partial_N h_M{}^A - (E_N{}^C h_M{}^{B'} - E_M{}^C h_N{}^{B'}) f_{B'C}{}^A - h_N{}^{C'} h_M{}^{B'} f_{B'C'}{}^A \quad (4.13)$$

and $h_{\underline{\mu}}{}^{A'}| = 0$, we see that $(\partial_{\underline{\nu}} h_{\underline{\mu}}{}^A + \partial_{\underline{\mu}} h_{\underline{\nu}}{}^A)|$ is written in terms of curvatures:

$$(\partial_{\underline{\mu}} h_{\underline{\nu}}{}^A + \partial_{\underline{\nu}} h_{\underline{\mu}}{}^A)| = \delta_{\underline{\mu}}{}^{\underline{\alpha}} \delta_{\underline{\nu}}{}^{\underline{\beta}} R_{\underline{\alpha}\underline{\beta}}{}^A|, \quad (4.14)$$

which means that $(\partial_{\underline{\nu}} h_{\underline{\mu}}{}^A + \partial_{\underline{\mu}} h_{\underline{\nu}}{}^A)|$ is written in the component language. Note also that the lowest components of curved indexed curvatures are always rewritten by flat indexed ones and can be expressed in terms of the fields in component approach as

$$R_{\underline{\nu}\underline{\mu}}{}^A| = \delta_{\underline{\nu}}{}^{\underline{\gamma}} \delta_{\underline{\mu}}{}^{\underline{\beta}} R_{\underline{\gamma}\underline{\beta}}{}^A|, \quad R_{\underline{\nu}m}{}^A| = \delta_{\underline{\nu}}{}^{\underline{\gamma}} e_m{}^b R_{\underline{\gamma}b}{}^A| - \frac{1}{2} \delta_{\underline{\nu}}{}^{\underline{\gamma}} \psi_m{}^{\underline{\beta}} R_{\underline{\gamma}\underline{\beta}}{}^A|. \quad (4.15)$$

Then the second order derivative $\partial_{\underline{\rho}}(\partial_{\underline{\nu}} h_{\underline{\mu}}{}^A + \partial_{\underline{\mu}} h_{\underline{\nu}}{}^A)|$ is similarly found to be written by the curvatures

$$\partial_{\underline{\rho}}(\partial_{\underline{\nu}} h_{\underline{\mu}}{}^A + \partial_{\underline{\mu}} h_{\underline{\nu}}{}^A)| = \partial_{\underline{\rho}} R_{\underline{\nu}\underline{\mu}}{}^A| + \frac{1}{2} (\delta_{\underline{\mu}}{}^{\underline{\gamma}} R_{\underline{\rho}\underline{\nu}}{}^{B'} + \delta_{\underline{\nu}}{}^{\underline{\gamma}} R_{\underline{\rho}\underline{\mu}}{}^{B'}) f_{B'\underline{\gamma}}{}^A|. \quad (4.16)$$

The first term in the r.h.s., the spinor derivative on curved indexed curvatures $\partial_{\underline{\rho}} R_{\underline{\nu}\underline{\mu}}{}^A|$ is nontrivial, but written by the covariant derivatives on flat indexed curvatures as

$$\begin{aligned} \partial_{\underline{\rho}} R_{\underline{\nu}\underline{\mu}}{}^A| &= \partial_{\underline{\rho}} (E_{\underline{\nu}}{}^C E_{\underline{\mu}}{}^B R_{CB}{}^A)| \\ &= \frac{1}{2} R(P)_{\underline{\rho}\underline{\nu}}{}^C \delta_{\underline{\mu}}{}^{\underline{\beta}} R_{C\underline{\beta}}{}^A| - \frac{1}{2} \delta_{\underline{\nu}}{}^{\underline{\gamma}} R(P)_{\underline{\rho}\underline{\mu}}{}^B R_{\underline{\gamma}B}{}^A| + \delta_{\underline{\nu}}{}^{\underline{\gamma}} \delta_{\underline{\mu}}{}^{\underline{\beta}} \delta_{\underline{\rho}}{}^{\underline{\delta}} \nabla_{\underline{\delta}} R_{\underline{\gamma}\underline{\beta}}{}^A|. \end{aligned} \quad (4.17)$$

Here we have used the definition of (the lowest component of) the torsion $R(P)_{\rho\nu}{}^C| = (\partial_{\rho} E_{\nu}{}^C + \partial_{\nu} E_{\rho}{}^C)|$ and the fact that $\partial_{\underline{\mu}}$ in the present gauge is equal to $\delta_{\underline{\mu}}{}^{\underline{\alpha}} \nabla_{\underline{\alpha}}$ at the lowest level, that is,

$$\partial_{\underline{\mu}} \Phi| = (\partial_{\underline{\mu}} - h_{\underline{\mu}}{}^{A'} X_{A'}) \Phi| = \delta_{\underline{\mu}}{}^{\underline{\alpha}} \nabla_{\underline{\alpha}} \Phi| \quad (4.18)$$

on a covariant quantity Φ . Therefore $\partial_{\underline{\rho}}(\partial_{\underline{\nu}} h_{\underline{\mu}}{}^A + \partial_{\underline{\mu}} h_{\underline{\nu}}{}^A)|$ is written in terms of the fields in component approach, and so are all the second order derivative of the spinor gauge fields.

The same is true for the third order derivative $\partial_{\underline{\sigma}} \partial_{\underline{\rho}}(\partial_{\underline{\nu}} h_{\underline{\mu}}{}^A + \partial_{\underline{\mu}} h_{\underline{\nu}}{}^A)|$. This is also written as

$$\partial_{\underline{\sigma}} \partial_{\underline{\rho}}(\partial_{\underline{\nu}} h_{\underline{\mu}}{}^A + \partial_{\underline{\mu}} h_{\underline{\nu}}{}^A)| = \partial_{\underline{\sigma}} \partial_{\underline{\rho}} R_{\underline{\nu}\underline{\mu}}{}^A| - \partial_{\underline{\sigma}} \partial_{\underline{\rho}} ((E_{\underline{\mu}}{}^C h_{\underline{\nu}}{}^{B'} + E_{\underline{\nu}}{}^C h_{\underline{\mu}}{}^{B'}) f_{B'C}{}^A + h_{\underline{\mu}}{}^{C'} h_{\underline{\nu}}{}^{B'} f_{B'C'}{}^A)|. \quad (4.19)$$

The terms other than $\partial_{\underline{\sigma}} \partial_{\underline{\rho}} R_{\underline{\nu}\underline{\mu}}{}^A|$ contain at most second order derivatives of gauge fields and can be written in component language as shown above. The non-trivial term $\partial_{\underline{\sigma}} \partial_{\underline{\rho}} R_{\underline{\nu}\underline{\mu}}{}^A|$ is expanded as

$$\partial_{\underline{\sigma}} \partial_{\underline{\rho}} R_{\underline{\nu}\underline{\mu}}{}^A| = \delta_{\underline{\sigma}}{}^{\underline{\delta}} \delta_{\underline{\rho}}{}^{\underline{\gamma}} \delta_{\underline{\nu}}{}^{\underline{\beta}} \delta_{\underline{\mu}}{}^{\underline{\alpha}} \nabla_{\underline{\delta}} \nabla_{\underline{\gamma}} R_{\underline{\beta}\underline{\alpha}}{}^A| + \cdots, \quad (4.20)$$

where \dots denotes the terms which are written by the curvatures and their first order covariant spinor derivatives with coefficients of at most second order derivatives of the gauge fields. Thus the second order spinor derivatives of the curved indexed curvatures and hence all the third order derivatives of the gauge fields are shown to be written in terms of the known fields in component approach. The third order curved spinor derivatives on curved indexed curvatures is similarly given by at most third order flat indexed covariant spinor derivatives on flat indexed curvatures. Thus, we have shown all orders of derivative of curved spinor gauge superfields are written in terms of the known fields in component approach.

The above procedure also makes sense for the gauge fixing in Poincaré superspace approach to the component SUGRA (the route III in Fig. 1), since we only use the general definitions of curvatures and inhomogeneous terms in the gauge transformation laws. The only difference is the constraints on curvatures, i.e., their explicit forms after gauge fixing.

We comment on other possible gauge-fixing conditions. In the treatment of Ref. [10], the dotted spinor gauge fields are gauge-fixed at superfield level to be $E^{\dot{\mu}A} = \delta^{\dot{\mu}}_{\dot{\alpha}}$ and $h^{\dot{\mu}A'} = 0$. In this case, $\xi^{(1,1)A\dot{\mu}}_{\mu}$ is used for realizing $\partial_{\mu}h^{\dot{\mu}A}| = 0$, and $\xi^{(2,1)A\dot{\mu}}$ for $\partial^2 h^{\dot{\mu}A}| = 0$, and so on. On the other hand, the undotted spinor gauge superfields h_{μ}^A generally remain unfixed. Another different example of gauge fixing will be discussed in section 5 for the superspace counterpart of the improved gauge in component approach.

4.2 Higher θ components of vector gauge superfields

In the previous section we have shown that all the higher θ components of spinor gauge superfields are properly gauge-fixed, and have already used all the θ components of the gauge transformation parameters other than the lowest. But there still exist the vector gauge superfields to be gauge-fixed for going down to the tensor calculus. So it is non-trivial whether all the higher θ components of vector gauge superfields are written in terms of known fields in component approach. By using the gauge conditions (4.3) and the definition of curvature $R_{\underline{\mu}n}^A$ in Eq. (4.13), we find $\partial_{\underline{\nu}}h_n^A$ is written as

$$\begin{aligned}\partial_{\underline{\mu}}h_n^A| &= \delta_{\underline{\mu}}^{\underline{\delta}}E_n^E R_{\underline{\delta}E}^A| - \delta_{\underline{\mu}}^C h_n^{B'}|f_{B'C}^A \\ &= \delta_{\underline{\mu}}^{\underline{\delta}}e_n^e R_{\underline{\delta}e}^A| + (-)^1 \delta_{\underline{\mu}}^{\underline{\delta}1} \psi_n^{\underline{\epsilon}} R_{\underline{\delta}\underline{\epsilon}}^A| - \delta_{\underline{\mu}}^{\underline{\gamma}} h_n^{B'}|f_{B'\underline{\gamma}}^A.\end{aligned}\quad (4.21)$$

This shows that the first order spinor derivative is expressed in terms of the fields in component approach. The analysis is performed in similar ways for the other higher components. For example, using Eq. (4.13), we obtain

$$\begin{aligned}\partial_{\underline{\rho}}\partial_{\underline{\mu}}h_n^A| &= \partial_{\underline{\rho}}R_{\underline{\mu}n}^A| + \partial_n\partial_{\underline{\rho}}h_{\underline{\mu}}^A| \\ &+ ((\partial_{\underline{\rho}}E_n^C)h_{\underline{\mu}}^{B'} + E_n^C(\partial_{\underline{\rho}}h_{\underline{\mu}}^{B'}) - (\partial_{\underline{\rho}}E_{\underline{\mu}}^C)h_n^{B'} - E_{\underline{\mu}}^C(\partial_{\underline{\rho}}h_n^{B'}))|f_{B'C}^A \\ &+ (\partial_{\underline{\rho}}h_n^{C'})h_{\underline{\mu}}^{B'}|f_{B'C'}^A + h_n^{C'}(\partial_{\underline{\rho}}h_{\underline{\mu}}^{B'})|f_{B'C'}^A.\end{aligned}\quad (4.22)$$

The spinor derivative of curved indexed curvatures can be replaced by its covariant derivative,

$$\begin{aligned}\partial_{\underline{\mu}}R_{\underline{\nu}p}^A| &= \partial_{\underline{\mu}}E_{\underline{\nu}}^C E_p^B R_{CB}^A| \\ &= (\partial_{\underline{\mu}}E_{\underline{\nu}}^C)E_p^B R_{CB}^A| - E_{\underline{\nu}}^C(\partial_{\underline{\mu}}E_p^B)R_{CB}^A| - E_{\underline{\nu}}^C E_p^B \delta_{\underline{\mu}}^{\underline{\delta}} \nabla_{\underline{\delta}} R_{CB}^A|.\end{aligned}\quad (4.23)$$

Therefore the second order spinor derivative of vector gauge superfields are found to be written by the curvatures, their covariant derivatives, and the lowest components of curved vector superfields. For higher θ terms, we need to evaluate higher order spinor derivatives on curvatures which become

$$\partial_{\underline{\mu}}\partial_{\underline{\nu}}R_{PQ}{}^{\mathcal{A}}| = \partial_{\underline{\mu}}\partial_{\underline{\nu}}E_P{}^CE_Q{}^DR_{CD}{}^{\mathcal{A}}| = E_P{}^CE_Q{}^DE_{\underline{\mu}}{}^GE_{\underline{\nu}}{}^F\nabla_G\nabla_FR_{CD}{}^{\mathcal{A}}| + \dots, \quad (4.24)$$

where \dots means the first order spinor derivatives on curvatures and/or the second order derivatives on vector gauge superfields. These have already been shown to be expressed by component approach fields. The same is clearly true for the third order derivative of curvatures. In the end, under the gauge conditions (4.3), all order θ components in curved vector and spinor gauge superfields are written in the language of component approach, and hence properly gauge-fixed.

4.3 Higher θ gauge invariance in component approach

We have shown in superspace approach that all the extra fields other than those appearing in component approach can be eliminated by using the higher θ gauge degrees of freedom, $\xi^{(n,m)\mathcal{A}}(x)$ ($n+m \geq 1$). We here add an interesting remark which might sound surprising:

All the fields appearing in component approach are in fact higher θ gauge invariant, i.e., invariant under the gauge transformation with parameters $\xi^{(n,m)\mathcal{A}}(x)$ ($n+m \geq 1$) but with $\xi^{(0,0)\mathcal{A}}(x) = 0$.

The gauge fields in component approach correspond to the lowest components of the gauge superfields with *vector* index $h_m{}^{\mathcal{A}}|$, like the vierbein $E_m{}^a| = e_m{}^a$ and the gravitino $E_m{}^{\underline{\alpha}}| = \psi_m{}^{\underline{\alpha}}$. Taking the lowest component of the general gauge transformation law (4.1) for the vector index case $\underline{\mu} \rightarrow m$, we have

$$\delta_G(\xi^{\mathcal{B}}X_{\mathcal{B}})h_m{}^{\mathcal{A}}| = \partial_m\xi^{\mathcal{A}}| + h_m{}^{\mathcal{C}}\xi^{\mathcal{B}}|f_{\mathcal{B}\mathcal{C}}{}^{\mathcal{A}}. \quad (4.25)$$

This contains only the lowest component of gauge parameter $\xi^{\mathcal{A}}| = \xi^{(0,0)\mathcal{A}}(x)$ and its spacial derivative, and does not involve any higher θ gauge parameter $\xi^{(n,m)\mathcal{A}}(x)$ ($n+m \geq 1$). Therefore the gauge fields in component approach are higher θ gauge invariant.

Furthermore, *all component fields of matter multiplets are also higher θ gauge invariant*, provided that they are identified with the lowest components of the *covariant derivatives* of matter superfields. For instance, the component fields in a general matter multiplet $[\mathcal{C}, \mathcal{Z}, \mathcal{H}, \dots]$ in component approach are expressed by using the covariant derivatives of a primary superfield Φ as [1]

$$\mathcal{C} = \Phi|, \quad \mathcal{Z}_{\alpha} = -i\nabla_{\alpha}\Phi|, \quad \mathcal{H} = \frac{1}{4}(\nabla^2\Phi + \bar{\nabla}^2\Phi)|, \quad \dots. \quad (4.26)$$

Since these covariant derivative quantities are literally covariant, their gauge transformation contain no derivative of the gauge parameter superfield $\xi^{\mathcal{A}}(x, \theta, \bar{\theta})$ so that their lowest components contain only the lowest component $\xi^{\mathcal{A}}| = \xi^{(0,0)\mathcal{A}}$, e.g. for the first derivative,

$$\delta(\xi^{\mathcal{A}}P_A)\nabla_{\alpha}\Phi| = \xi^{\mathcal{A}}|\nabla_A(\nabla_{\alpha}\Phi)|, \quad (4.27)$$

and hence higher θ gauge invariant.

In this sense, the invariant action formulas in component approach are not only invariant under the usual gauge transformation with the parameter $\xi^{\mathcal{A}}(x) = \xi^{(0,0)\mathcal{A}}$, but fully gauge invariant with the superfield parameter $\xi^{\mathcal{A}}(x, \theta, \bar{\theta})$. For instance, consider the F-type action formula of conformal SUGRA [6, 10]

$$S_F^{\text{comp}} = \int d^4x e \left(-\frac{1}{4} \nabla^2 W + \frac{i}{2} \bar{\psi}_{a\dot{\alpha}} (\bar{\sigma}^a)^{\dot{\alpha}\beta} \nabla_{\beta} W - (\bar{\psi}_a \bar{\sigma}^{ab} \bar{\psi}_b) W \right) \Big| + \text{h.c.} \quad (4.28)$$

The integrands is written in terms of the vierbein e_m^a , the gravitino ψ_m^{α} , and the lowest components of the covariant superfields, $W|$, $\nabla_a W|$, $\nabla^2 W|$. All of these quantities are higher θ gauge invariant, and hence the action is fully gauge invariant with superfield transformation parameters.

Indeed the above component expression for the F-type action formula was derived in [10] starting from the superspace action

$$S_F^{\text{SS}} = \int d^4x d^2\theta \mathcal{E} W + \text{h.c.}, \quad (4.29)$$

which has the manifest superfield gauge invariance. While his derivation uses a particular gauge fixing for the higher θ gauge symmetry, the final component action S_F^{comp} (4.28) is also superfield gauge invariant. Both expressions S_F^{comp} and S_F^{SS} are fully superfield gauge invariant and coincide with each other in a particular gauge, so implying that they coincide in any gauge.

5 Correspondence of superconformal gauge fixing

In this section, we clarify more explicitly the correspondence of superconformal gauge fixing between the superspace and component approaches. In component approach, the gauge-fixing condition that leads to the canonically normalized EH and RS terms was firstly given by Kugo-Uehara (KU) in Ref. [7], called the KU gauge in what follows.

5.1 Correspondence of YM sectors

The following is the translational dictionary for the YM sector in the YM matter coupled conformal SUGRA between the superspace formulation in section 2 and the component one in Ref. [7].

component	superspace
$B_{\mu}^{\alpha}, F_{\mu\nu}^{\alpha}, \hat{F}_{ab}^{\alpha}$	$\mathcal{A}_m^{(a)} , \mathcal{F}_{mn}^{(a)} , \mathcal{F}_{ab}^{(a)} $
$W_R^{\alpha}, W_L^{\alpha}$	$\mathcal{W}_{\alpha}^{(a)} , -\mathcal{W}^{(a)\dot{\alpha}} $
$D^{\alpha}, f_{\alpha\beta}$	$\frac{i}{2} \nabla^{\alpha} \mathcal{W}_{\alpha}^{(a)} = \frac{i}{2} \bar{\nabla}_{\dot{\alpha}} \mathcal{W}^{(a)\dot{\alpha}} , H_{(a)(b)} $
$iT^{\alpha}{}^j{}_j z_j, -iz^{*j} T^{\alpha}{}_{\dot{j}}{}^i$	$V_{(a)}^i , \bar{V}_{(a)}^{i*} $

(5.1)

In this dictionary, we have rescaled the gauge fields and the gaugino multiplet of internal symmetry by the gauge coupling \tilde{g} such as $\tilde{g}B_\mu^\alpha \rightarrow B_\mu^\alpha$. In Ref. [7], the internal symmetry was discussed only for the linear case where the Killing vectors are given by the representation matrices. More general cases of Killing vectors which preserve the Kähler structure were constructed in the framework of superconformal tensor calculus [9]. With the same form of Kähler potential, our $V_{(a)}^i$, $\bar{V}_{(a)}^{i*}$, $F_{(a)}$ and $J_{(a)}$ correspond to $k_{\alpha i}$, k_α^i , $3r_\alpha$ and $\mathcal{P}_\alpha(z, z^*)$ in Ref. [9], respectively.

5.2 Non-vanishing superpotential case

As in the system without YM gauge fields, the gauge-fixing conditions in superspace (3.3) are found to correspond to the KU gauge:

$$\begin{aligned} z_0 = \sqrt{3}\phi^{-\frac{1}{2}}(z, z^*) = e^{-\mathcal{G}/6} &\leftrightarrow \Phi^0| = e^{G/6}|, \\ \chi_{R0} = -z_0\phi^{-1}\phi^i\chi_{Ri} = -\frac{1}{3}e^{-\mathcal{G}/6}\mathcal{G}^i\chi_{Ri} &\leftrightarrow \nabla_\alpha\Phi^0| = \frac{1}{3}e^{G/6}G_i\nabla_\alpha\Phi^i|, \end{aligned} \quad (5.2)$$

where z_0 and χ_{R0} are the lowest and spinor components of the chiral compensator in the superconformal tensor calculus, and we have used the correspondence of conformal multiplets, developed in [1]. Unlike Ref. [1], the covariant derivative ∇_α now contains the gauge field of internal symmetry.

Among various correspondences, we here focus on the correspondence of the Poincaré supersymmetry after gauge fixing. Since the spinor derivatives are identified with the supersymmetry transformations both in the conformal and Poincaré superspaces, Eq. (3.6) leads to the Poincaré supersymmetry transformation defined by the superconformal gauge transformations

$$\delta_G(\eta^\alpha Q_\alpha^P) = \delta_G(\eta^\alpha Q_\alpha) + \delta_G(\xi(A)'(\eta)A) + \delta_G(\xi(K)'(\eta)^B K_B) \quad (5.3)$$

with the transformation parameters $\xi(A)'(\eta) = \eta^\alpha A_\alpha + \bar{\eta}_{\dot{\alpha}} A^{\dot{\alpha}}$ and $\xi(K)'(\eta)^A = \eta^\beta f_\beta^A + \bar{\eta}_{\dot{\beta}} f^{\dot{\beta}A}$. Using the explicit form of the spinor gauge fields A_α obtained in (3.5), we have the A -transformation parameter

$$\xi(A)'(\eta)| = \frac{i}{4}G_j\eta^\alpha\mathcal{D}_\alpha^P\Phi^j| - \frac{i}{4}G_{j*}\bar{\eta}_{\dot{\alpha}}\bar{\mathcal{D}}^{P\dot{\alpha}}\bar{\Phi}^{j*}|, \quad (5.4)$$

which is the exactly same form as the previous result in the system without YM (Eq. (4.28) in [1]), though the covariant derivatives in the above contain the YM covariantization terms. The spinor K_A -gauge fields f_α^A under the gauge fixing (3.3) are also evaluated by using (3.4) and (3.15), and the resultant transformation parameters are given by

$$\begin{aligned} \xi(K)'(\eta)_\alpha| &= -\frac{1}{8}\left(e^{-G/6}\nabla^2\Phi^0 - \frac{1}{3}G_i\nabla^2\Phi^i\right)\eta_\alpha| \\ &\quad - \frac{1}{12}\left(\left(G_{ij} + \frac{1}{3}G_iG_j\right)\eta^\gamma\nabla_\gamma\Phi^j + G_{ij*}\bar{\eta}_{\dot{\beta}}\bar{\nabla}^{\dot{\beta}}\bar{\Phi}^{j*}\right)\nabla_\alpha\Phi^i| \\ &\quad - \frac{1}{12}e_c^m\left(G_i\nabla_m\Phi^i - G_{i*}\nabla_m\bar{\Phi}^{i*}\right)i\sigma_{\alpha\dot{\beta}}^c\bar{\eta}^{\dot{\beta}}| - \frac{i}{3}e_c^mA_m i\sigma_{\alpha\dot{\beta}}^c\bar{\eta}^{\dot{\beta}}|, \end{aligned} \quad (5.5)$$

$$\begin{aligned} \xi(K)'(\eta)_a| &= -e_a{}^m(f_m{}^\beta\eta_\beta + f_{m\dot{\beta}}\bar{\eta}^{\dot{\beta}}) \\ &\quad + \frac{1}{2}e_a{}^m\psi_m{}^\alpha(\eta^\beta f_{\beta\alpha} + \bar{\eta}_{\dot{\beta}}f^{\dot{\beta}}{}_\alpha)| + \frac{1}{2}e_a{}^m\bar{\psi}_{m\dot{\alpha}}(\eta^\beta f_{\beta}{}^{\dot{\alpha}} + \bar{\eta}_{\dot{\beta}}f^{\dot{\beta}\dot{\alpha}})|. \end{aligned} \quad (5.6)$$

These are also the same expressions as in Ref. [1]. Thus the transformation parameters (5.4)-(5.6) are found to agree with those in the superconformal tensor calculus (Eqs. (40), (42a), (42b) in [7]).

At this moment, one question arises. When we go down from the conformal superspace to the component superconformal tensor calculus, the lowest components of curved spinor gauge superfields are gauge-fixed to zero, $h_\mu{}^{\mathcal{A}}| = 0$, other than the vierbein. This means that the lowest components of flat spinor gauge superfields are also equal to zero, that is, $h_\mu{}^{\mathcal{A}}| = \delta_\mu{}^\alpha h_\alpha{}^{\mathcal{A}}| = 0$. However when we use the superspace version of the KU gauge, non-vanishing flat spinor gauge superfields A_α and $f_\alpha{}^A$ appear and correspond to the A and K_A compensations in the Poincaré supersymmetry transformation in component approach as seen above. Why does the component approach know non-vanishing spinor gauge fields? What is the origin of them?

The answer is the resetting of gauge conditions. Let us consider the A gauge superfields as an example. When going down to the component superconformal tensor calculus, all the higher components of gauge parameter superfields other than the lowest are used to fix the superfield A_μ and, in particular, its lowest component $A_\mu| = \delta_\mu{}^\alpha A_\alpha|$ is set equal to zero by using the parameter $\partial_\mu \xi(A)|$. On the other hand, when going down to the Poincaré superspace, the whole gauge parameter superfields are used for $\xi(A)$, $\xi(D)$ and $\xi(K)$ to realize different gauge-fixing conditions. For example, $\xi(A)$ is chosen as $\xi(A) = \frac{3}{4i} \log(\bar{\Phi}^0/\Phi^0)$ to have the KU gauge counterpart in the superspace [1]. This form of gauge parameter superfield resets its component $\partial_\mu \xi(A)|$ to a different value from the above, and therefore $A_\mu|$ is generally reset to a nonzero quantity. In particular, the form of spinor A -gauge field should coincide with the one appearing in the A -gauge compensation term in the Poincaré supersymmetry transformation, since the supersymmetry transformation after gauge fixing is unique.

We show this resetting explicitly. When we perform the above A -gauge transformation with the parameter $\xi(A) = \frac{3}{4i} \log(\bar{\Phi}^0/\Phi^0)$, the spinor gauge field $A_\mu|$ is reset from the initial value $A_\mu| = 0$ as

$$A_\mu| = 0 + \frac{3}{4i} \partial_\mu \log(\bar{\Phi}^0/\Phi^0)| = \frac{3i}{4} \frac{1}{\Phi^0} \delta_\mu{}^\alpha \nabla_\alpha \Phi^0|. \quad (5.7)$$

Plugging the expressions $\Phi^0 = e^{G/6}$ and $\nabla_\alpha \Phi^0| = \frac{1}{3} e^{G/6} G_i \mathcal{D}_\alpha^P \Phi^i|$ in the KU gauge, we find

$$A_\alpha| = \frac{i}{4} G_i \mathcal{D}_\alpha^P \Phi^i|, \quad (5.8)$$

which exactly reproduces $A_\alpha|$ used for the gauge parameter $\xi(A)'(\eta)$ in (5.4). A similar discussion is possible for the K_A gauge fields, but more complicated. What KU found is that one can avoid explicit and complicated computations of gauge transformations. The final expression of the spinor gauge superfields can be found by identifying the Poincaré supersymmetry transformation with the one which retains the D , A , K_A gauge-fixing conditions.

5.3 Vanishing superpotential case

The gauge-fixing condition (3.9) for the case of vanishing superpotential is related to the formulation of Ref. [9] in component approach and also to the isometric Kähler superspace of Ref. [14]. This can be seen in the same way as discussed before by using Eqs. (3.9) and (3.15). In particular, the transformation parameter of the resultant Poincaré supersymmetry in Eq. (5.3) becomes

$$\xi(A)'(\eta) = \eta^{\underline{\alpha}} \left(A_{\underline{\alpha}} - \frac{i}{4} (F_{(a)} - \bar{F}_{(a)}) \mathcal{A}_{\underline{\alpha}} \right) = \frac{i}{4} K_j \eta^{\alpha} \tilde{\mathcal{D}}_{\alpha}^P \Phi^j - \frac{i}{4} K_{j*} \bar{\eta}_{\dot{\alpha}} \tilde{\bar{\mathcal{D}}}^{P\dot{\alpha}} \bar{\Phi}^{j*}, \quad (5.9)$$

which is the same form as (5.4) with a replacement $G \rightarrow K$. The other parameters $\xi(K)'(\eta)^A$ are also given by the same expressions as (5.5) and (5.6), with the same replacement $G \rightarrow K$ being performed.

6 Summary

In this paper, we have discussed the YM matter coupled conformal SUGRA in superspace and compare it with the component approach. We have introduced the YM gauge superfield of internal symmetry in conformal superspace by gauging the isometry of the Kähler manifold spanned by chiral matter superfields. The superconformal property of the gaugino superfield is derived by the Bianchi/Jacobi identities in Eq. (2.13). The YM gauge transformation laws of Kähler potential, superpotential and compensator are studied from the superspace viewpoint in Eqs. (2.19) and (2.21).

In section 3, we have presented the superspace gauge-fixing conditions leading to the canonically normalized EH and RS terms, which conditions give the KU-gauge counterpart in superspace. The relation between the Poincaré and conformal supersymmetry transformations (the covariant derivatives) is discussed at superfield level. In section 4 we have also shown the gauge-fixing procedure in detail how the conformal superspace formulation is reduced to the component approach (superconformal tensor calculus).

In section 5, the KU gauge in component approach is shown to be equivalent to the superspace gauge (3.3) written in terms of superfields. Then the relations between the Poincaré and conformal supersymmetry transformations are found to exactly correspond to each other in superspace and component approaches. Finally, we discuss several approaches with the canonically normalized EH and RS terms in the conformal superspace viewpoint.

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